Enclosed as requested is the report on the Vernal, Utah pipeline.
a) Determine the cost of running the pipeline strictly on BLM ground with two different scenarios:

i) Heading east through the mountain and then south to the refinery

$$
\text { Cost }=50,000 \$ *(3)+100,000 \$+500,000 \$+(25) * 300,000 \$=8,250,000 \$
$$

This is $50,000 \$$ every month for three months of delays plus the cost of $100,000 \$$ for the environmental study plus the cost of $500,000 \$$ to drill through the mountain plus the cost of running the pipeline for 25 miles at $300,000 \$$ per mile on BLM land.
ii) Running west, south and then east to the refinery.

Cost $=(27) * 300,000 \$=8,100,000 \$$
This is the cost of running the pipeline 27 miles at $300,000 \$$ per mile on BLM land.
b) Determine the cost of running the pipeline the shortest distance (straight line joining well to refinery across the private ground).

$D=\sqrt{20^{2}+5^{2}}=5 \sqrt{17} \approx 20.62$ miles D is the pipe line
Cost $=500,000 \$ * D(5 \sqrt{17})=2,500,000 \sqrt{17}=10,307,764.06 \$$

D is the distance from the well to the refinery through private ground. The cost is the distance through the private ground D at $500,000 \$$ for every mile.
c) Determine cost function and the optimal placement to minimize cost then sketch. Draw a graph of the function and label the point of minimum cost.

$x=$ horizontal distance from point across from well to the point where $H$ meets the BLM ground.
$\mathrm{H}=$ the distance of the pipe line across the private ground.
$\mathrm{L}=$ the distance of the pipeline from where it connects at the end of H to the refinery.

$$
\begin{aligned}
& H^{2}=x^{2}+5^{2} \rightarrow H=\sqrt{x^{2}+25} \quad L=20-x \\
& C(x)=500,000 \$ * H+300,000 * L \\
& C(x)=500,000 \$ *\left(\sqrt{x^{2}+25}\right)+300,000 *(20-x)
\end{aligned}
$$

This is the cost function, 500,000 dollars for every mile of H plus 300,000 dollars for every mile of L .

## Graph for 500000*sqrt(x^2+25)+300000*(20-x)


$C^{\prime}(x)=500,000 \$ *\left[\frac{1}{2}\left(x^{2}+25\right)^{-\frac{1}{2}} *[2 x]\right]+300,000 \$[-1]$
$C^{\prime}(x)=500,000 \$ *\left[\frac{x}{\sqrt{x^{2}+25}}\right]-300,000 \$$
By setting the derivative of the cost function to zero the optimal X can be found to minimize cost.

$$
\begin{aligned}
& 500,000 \$ *\left[\frac{x}{\sqrt{x^{2}+25}}\right]-300,000 \$=0 \rightarrow 500,000 \$ *\left[\frac{x}{\sqrt{x^{2}+25}}\right]=300,000 \$ \\
& \frac{x}{\sqrt{x^{2}+25}}=\frac{300,000 \$}{500,000 \$} \rightarrow \frac{x}{\sqrt{x^{2}+25}}=\frac{3}{5} \rightarrow x=\frac{3}{5} \sqrt{x^{2}+25}
\end{aligned}
$$

$\rightarrow x^{2}=\frac{9}{25} *\left(x^{2}+25\right) \rightarrow x^{2}=\frac{9 x^{2}}{25}+9 \quad \rightarrow \frac{16}{25} x^{2}=9 \rightarrow X^{2}=\frac{225}{16}$
$\rightarrow \quad x=\frac{15}{4}=3.75$ miles
Since the optimizing $x$ is found to find the minimum cost find $H$ and $L$ with the $x$ value $H=\sqrt{\frac{225}{16}+25} \rightarrow H=\sqrt{\frac{625}{16}} \rightarrow H=\frac{25}{4}=6.25$ miles
$L=20-\frac{15}{4}=\frac{65}{4}=16.25$ miles
$\sin (\theta)=\frac{x}{H} \quad \rightarrow \quad \sin (\theta)=\frac{\frac{15}{4}}{\frac{25}{4}}=\frac{3}{5}=.6 \rightarrow \theta=\sin ^{-1}(.6) \approx 36.87^{\circ}$
$\theta-90 \approx 53.13^{\circ}$

Then to find minimum cost input the value of $x$ into $C(x)$ or the $H$ and $L$ value.
$C(x)=500,000 \$ *(6.25)+300,000 *(16.25)=8,000,000 \$$


I have learned how to take limits of various kinds, I have learned to take derivatives of functions and implicit derivatives, I have learned to graph based on derivatives, and I have learned to take average and instant velocities, optimization based on derivatives among other things. Possible real world applications could be optimizing the speed to drive a long distance for gas mileage, minimizing the amount of wood in a construction project or finding out how fast the water drains out of an AR- 15 when it is submerged under water. Calculus is a very useful tool in physics and optimization and finance. While not much calculus is used in computer science it allows me to think logically and understand complex algorithms. When I am trying to create a proof I am exercising the same logical analysis and processes that I need to create a complex regular expression in programs and create algorithms and pseudo code to develop elaborate programs. Overall I think taking calculus and other pure mathematics is essential for the process of software development and engineering.

